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# Detuning effect in quantum dynamics of a strongly coupled single quantum dot–cavity system

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**Abstract**

The quantum dynamics of a strongly coupled single quantum dot–cavity system with non-zero detuning in a phonon bath is investigated theoretically in terms of a perturbation treatment based on a unitary transformation and an operator displacement. The decoherence due to phonons as a function of the detuning between the cavity mode and exciton is obtained analytically. It is shown that the detuning has a significant impact on the quantum dot exciton lifetime. In realistic experimental conditions, the calculated exciton lifetimes are in good agreement with recent experimental observation (Hennessy *et al* 2007 *Nature* **445** 896).

## 1. Introduction

The potential applications of semiconductor quantum dots confined in a microcavity in quantum information processing have generated considerable research efforts in recent years [1]. In such dot–cavity systems, excitons in quantum dots constitutes an alternate two-level system instead of the usual two-level atomic system. The nanocavity with a small volume and a high $Q$ is fabricated by micropillar, microdisk and a photonic crystal slab [1]. Recently, many experimental studies concentrate on vacuum Rabi splittings in these quantum dot–cavity systems [2–6]. Badolato *et al* [5] have used a deterministic approach to the implementation of solid-state cavity quantum electrodynamics (QED) based on a precise spatial and spectral overlap between a single quantum dot and a photonic crystal nanocavity. For such a quantum dot–cavity QED, the exciton–phonon interaction is very important during their interaction with photons [7, 8] and can cause decoherence. The understanding of decoherence due to phonons in these systems is also indispensable for practical applications. In terms of a polaron operator technique, Wilson-Rae and Imamoglu [9] have shown the reduction of dot–cavity coupling strength due to the exciton–phonon interaction. Zhu *et al* [10] have studied the impact of exciton–phonon coupling on a coherently driven dot–cavity system at zero detuning. The collapse and revival of Rabi oscillations are predicted. Larson *et al* [11] have also investigated Rabi oscillations in a quantum dot–cavity system coupled to a non-zero temperature phonon bath beyond the Born–Markov approximation. The influence of quantum lattice fluctuations on the vacuum Rabi frequency has been presented by Zhu and coauthors [12]. With density matrix theory and quantum kinetic equations, the phonon-assisted damping of Rabi oscillation has been investigated by Förstner *et al* [13, 14]. Utilizing the independent boson model, excitation transfer in coupled nanostructures has been solved exactly by Richter *et al* [15]. However the detuning effect between the exciton and cavity in such a system has not yet been shown. How to obtain the quantum dynamics of a strongly coupled quantum dot–cavity system including both the detuning effect and exciton–phonon coupling is difficult in both theory and experiment. Most recently, Hennessy *et al* [16] have investigated the quantum nature of a strongly coupled single quantum dot–cavity system with different detunings. The experimental results showed that the larger the detuning, the longer the quantum dot exciton lifetime. In theory, a generalized version of the rotating-wave approximation for the single-mode spin-boson model is presented for all values of the coupling strength and a wide range of detuning values [17]. On the other hand, Rabi oscillations of excitons in single quantum dots have been observed in recent experiments [18–22]. Especially, Ramsay *et al* [23] observed a damped Rabi oscillation in a coherently driven InGaAs quantum dot without cavity for several detunings. Wu *et al* [24] and Mogilevtsev *et al* [25] have theoretically demonstrated the damping of Rabi oscillations with a driving field in a quantum dot without cavity, which is a consequence of non-Markovian effects due to the coupling of the system to a phonon bath. The observation of such oscillations is a first step towards quantum information processing. In the present paper, we investigate the quantum dynamics of a dot–cavity system in the presence of decoherence due to a phonon bath. By using a variational approach, applying canonical transformations and using perturbation theory, we calculate the phonon induced decoherence rate and the survival probability of the initial state. We extend our previous model [10] to include detuning between the cavity mode and the exciton frequency since the detuning effects are important for the experiment by Hennessy *et al* [16]. The decoherence rate induced by acoustic phonons as a function of the detuning between the cavity mode and exciton is obtained explicitly and a comparison with recent experimental results is demonstrated. It is shown that the calculated values of quantum dot exciton lifetime through the relation to the decoherence rate are in agreement with the observed ones for non-zero detunings [16]. Further, the influence of the dot size on the decoherence rate is also discussed.

The paper is organized as follows. Section 2 gives the theoretical model for a single quantum dot–cavity system and solves it in terms of a perturbation method based on a unitary transformation and an operator displacement. In section 3, the numerical results for the phonon induced decoherence are shown and a comparison with the recent experimental result is presented. Finally, a summary is given in section 4.

## 2. Theory

Based on the recent experiment by Hennessy *et al* [16], we assume a single quantum dot confined in a single-mode microcavity and coupled to a phonon bath. The quantum dot is described by a conventional two-level model which consists of the electronic ground state $|g\rangle$ and the lowest electron–hole(exciton) state $|e\rangle$ [21, 26]. This simple model has been used widely and testified to be a successful one to explain the key nature of a single quantum dot [8, 9]. Then this single two-level system couples to a single cavity mode in the presence of decoherence due to a phonon bath. In such a way, we can treat this two-level system simply by pseudospin $-\frac{1}{2}$ operators $S^z$ and $S^{\pm}$. The cavity mode is characterized by the annihilation and creation operators $a$ and $a^+$, respectively. Such a dot–cavity system with a coherent drive at zero detuning is investigated by our previous work [10]. The collapse and revival of Rabi oscillations are predicted as the dot is coherently driven by a classical strong field, but there is no detuning effect in our previous work. Since the detuning effects are important for the experiment done by Hennessy *et al* [16], we should study quantum dynamics of the dot–cavity system with non-zero detuning in detail so that we can compare our calculations with the experimental results.

The Hamiltonian for this system in the rotating-wave approximation reads ($\hbar = 1$) [9, 10]:

$$H = \omega_{\rm ex} S^z + \omega_c a^+ a + g(S^+ a + S^- a^+) + \sum_{\vec q} \omega_{\vec q} c^+_{\vec q} c_{\vec q} + S^z \sum_{\vec q} M_{\vec q}(c^+_{\vec q} + c_{\vec q}). \tag{1}$$

This model Hamiltonian (equation (1)) is equivalent to the 'spin-boson' Hamiltonian coupled to a single cavity mode [9]. However, the 'spin-boson' Hamiltonian cannot be solved exactly. Various analytical or numerical approaches have been proposed to obtain an approximate solution of it (e.g. see Leggett *et al* [27]).

We apply a transformation $\exp[-{\rm i}\omega_c(S^z + a^+ a)t]$ to the above Hamiltonian in a rotating frame with cavity frequency $\omega_c$, and then the Hamiltonian is transformed to

$$H = \Delta S^z + g(S^+ a + S^- a^+) + \sum_{\vec q} \omega_{\vec q} c^+_{\vec q} c_{\vec q} + S^z \sum_{\vec q} M_{\vec q}(c^+_{\vec q} + c_{\vec q}), \tag{2}$$

where $\Delta = \omega_{\rm ex} - \omega_c$ is the detuning between the cavity mode and exciton with energy $\omega_{\rm ex}$. $g$ is the single-photon Rabi frequency. $c^+_{\vec q}$ ($c_{\vec q}$) is the creation (annihilation) operator of the phonon with the momentum $\vec q$ and energy $\omega_{\vec q}$. $M_{\vec q}$ refers to the matrix elements characterizing the exciton–phonon interaction. For simplicity, we ignore the off-diagonal exciton–phonon interaction if the energy spacing between the states in quantum dots is larger than 20 meV while the temperature is low enough ($T < 50$ K) [8]. The Hamiltonian (2) includes both non-zero detuning and exciton–phonon coupling and is not easy to solve even using the method shown in [10]. Therefore, we firstly make a displacement to all boson modes [28],

$$c_{\vec q} = b_{\vec q} - \frac{M_{\vec q}}{2\omega_{\vec q}} \sigma_0, \tag{3}$$

where $\sigma_0$ is a constant which can be determined later. Then we apply a canonical transformation [10, 29, 30],

$$H' = \exp(A) H \exp(-A) \tag{4}$$

with

$$A = \sum_{\vec q} \frac{M_{\vec q}}{\omega_{\vec q}} \xi_{\vec q} (b^+_{\vec q} - b_{\vec q})(S^z - \sigma_0/2), \tag{5}$$

where $\xi_{\vec q}$ is a variational parameter and the Hamiltonian is decomposed into three parts,

$$H' = H'_0 + H'_1 + H'_2 \tag{6}$$

where

$$H'_0 = \Delta' S^z + \eta g(S^+ a + S^- a^+) + \sum_{\vec q} \omega_{\vec q} b^+_{\vec q} b_{\vec q} - \sum_{\vec q} \frac{M^2_{\vec q}}{4\omega_{\vec q}} \xi_{\vec q}(2 - \xi_{\vec q}) + \sum_{\vec q} \frac{M^2_{\vec q}}{4\omega_{\vec q}} \sigma_0^2 (1 - \xi_{\vec q})^2, \tag{7}$$

$$H'_1 = \sum_{\vec q} M_{\vec q}(1 - \xi_{\vec q})(b^+_{\vec q} + b_{\vec q})(S^z - \sigma_0/2) + \eta g(S^+ a - S^- a^+) \sum_{\vec q} \frac{M_{\vec q}}{\omega_{\vec q}} \xi_{\vec q}(b^+_{\vec q} - b_{\vec q}), \tag{8}$$

$$H'_2 = g S^+ a \left[\exp\left(\sum_{\vec q} \frac{M_{\vec q}}{\omega_{\vec q}} \xi_{\vec q}(b^+_{\vec q} - b_{\vec q})\right) - \eta\right] + {\rm h.c.} - \eta g(S^+ a - S^- a^+) \sum_{\vec q} \frac{M_{\vec q}}{\omega_{\vec q}} \xi_{\vec q}(b^+_{\vec q} - b_{\vec q}), \tag{9}$$

where

$$\Delta' = \Delta - \tau\sigma_0, \qquad \tau = \sum_{\vec{q}} \frac{M_{\vec{q}}^2}{\omega_{\vec{q}}}(1-\xi_{\vec{q}})^2. \quad (10)$$

The variational parameters $\eta$ and $\xi_{\vec{q}}$ can be adjusted to minimize $H_1'$ and $H_2'$. Next we apply another canonical transformation [31]

$$H'' = \exp(Q)H'\exp(-Q) = H_0'' + H_1'' + H_2'' \quad (11)$$

with

$$Q = \theta(4N)^{-\frac{1}{2}}(S^+a - S^-a^+), \quad (12)$$

where

$$N = a^+a + S^z + \tfrac{1}{2}, \quad (13)$$

$$\cos\theta = \frac{\Delta'}{W}, \qquad \sin\theta = \frac{2\eta g\sqrt{N}}{W}, \quad (14)$$

$$W = \sqrt{4(\eta g)^2 N + (\Delta')^2}, \quad (15)$$

where $N$ is an operator that represents the total excitation in the exciton–cavity field system. We can easily obtain

$$H_0'' = WS^z + \sum_{\vec{q}} \omega_{\vec{q}} b_{\vec{q}}^+ b_{\vec{q}} - \sum_{\vec{q}} \frac{M_{\vec{q}}^2}{4\omega_{\vec{q}}}\xi_{\vec{q}}(2-\xi_{\vec{q}}) + \sum_{\vec{q}} \frac{M_{\vec{q}}^2}{4\omega_{\vec{q}}}\sigma_0^2(1-\xi_{\vec{q}})^2, \quad (16)$$

$$H_1'' = \sum_{\vec{q}} M_{\vec{q}}(1-\xi_{\vec{q}})(b_{\vec{q}}^+ + b_{\vec{q}})\left(\frac{\Delta'}{W}S^z - \sigma_0/2\right) - \frac{\eta g}{W}(S^+a + S^-a^+)\sum_{\vec{q}} M_{\vec{q}}(1-\xi_{\vec{q}})(b_{\vec{q}}^+ + b_{\vec{q}}) + \eta g(S^+a - S^-a^+)\sum_{\vec{q}} \frac{M_{\vec{q}}}{\omega_{\vec{q}}}\xi_{\vec{q}}(b_{\vec{q}}^+ - b_{\vec{q}}), \quad (17)$$

$$H_2'' = \exp(Q)H_2'\exp(-Q). \quad (18)$$

Obviously, due to the decoupling of the phonon subsystem and the exciton–photon subsystem, we can easily solve $H_0''$ for which the eigenstates can be expressed directly as $|\pm\rangle|\{n_{\vec{q}}\}\rangle$ with

$$|+\rangle = |e, m\rangle, \qquad |-\rangle = |\mathrm{g}, m\rangle, \quad (19)$$

where $m$ refers to the photon number of the cavity field while $|\{n_{\vec{q}}\}\rangle$ means that there are $n_{\vec{q}}$ phonons for mode $\vec{q}$. The ground state of $H_0''$ is given by

$$|G_0\rangle = |-\rangle|\{0_{\vec{q}}\}\rangle, \quad (20)$$

where $|\{0_{\vec{q}}\}\rangle$ represents the vacuum state for phonons. Since $H_1''$ and $H_2''$ should be small enough to be regarded as perturbation, we let $H_1''|G_0\rangle = 0$ and $\langle G_0|H_2''|G_0\rangle = 0$. Then we can obtain $\xi_{\vec{q}}$, $\sigma_0$ and $\eta$ as follows:

$$\xi_{\vec{q}} = \frac{\omega_{\vec{q}}}{\omega_{\vec{q}} + W}, \quad (21)$$

$$\sigma_0 = -\frac{\Delta'}{W}, \quad (22)$$

$$\eta = \exp\left(-\sum_{\vec{q}} \frac{M_{\vec{q}}^2}{2\omega_{\vec{q}}^2}\xi_{\vec{q}}^2\right). \quad (23)$$

We note that

$$\Delta' = \frac{\Delta}{1 - \frac{\tau}{W}}, \qquad \cos\theta = \frac{\Delta}{W-\tau}, \quad (24)$$

therefore $\theta$ is in the range of $0 \leqslant \theta \leqslant \pi/2$ and $\theta = \frac{\pi}{2}$ just corresponds to the case at resonance ($\Delta = 0$, i.e. $\omega_c = \omega_{ex}$).

Further, we denote the lowest excited states as $|+\rangle|\{0_{\vec{q}}\}\rangle$ and $|-\rangle|\{1_{\vec{q}}\}\rangle$ where $|\{1_{\vec{q}}\}\rangle$ means that there is only one phonon for mode $\vec{q}$ and no phonon for other modes. Then we can show that $\langle\{0_{\vec{q}}\}|\langle+|H_1''|-\rangle|\{1_{\vec{q}}\}\rangle = V_{\vec{q}}$, where $V_{\vec{q}} = -\frac{2\eta g\sqrt{m+1}M_{\vec{q}}\xi_{\vec{q}}}{\omega_{\vec{q}}}$. Next, we make linear combination of the lowest excited states of the $H_0''$ to diagonalize $H''$ as [29, 32]

$$H'' = -\frac{W}{2}|G_0\rangle\langle G_0| + \sum_E E|E\rangle\langle E|. \quad (25)$$

The experiment in [16] is performed at 4.2 K. At such a low temperature, the multiphonon process is weak enough to be negligible. So the transformation reads:

$$|-\rangle|\{1_{\vec{q}}\}\rangle = \sum_E y_{\vec{q}}(E)|E\rangle, \quad (26)$$

$$|+\rangle|\{0_{\vec{q}}\}\rangle = \sum_E x(E)|E\rangle, \quad (27)$$

$$|E\rangle = x(E)|+\rangle|\{0_{\vec{q}}\}\rangle + \sum_{\vec{q}} y_{\vec{q}}(E)|-\rangle|\{1_{\vec{q}}\}\rangle, \quad (28)$$

where

$$x(E) = \left[1 + \sum_{\vec{q}} \frac{V_{\vec{q}}^2}{\left(E + \frac{W}{2} - \omega_{\vec{q}}\right)^2}\right]^{-1/2}, \quad (29)$$

$$y_{\vec{q}}(E) = \frac{V_{\vec{q}}}{\left(E + \frac{W}{2} - \omega_{\vec{q}}\right)}x(E), \quad (30)$$

and the $E$'s are diagonalized excitation energies; they are the solutions of the equation

$$E - \frac{W}{2} - \sum_{\vec{q}} \frac{V_{\vec{q}}^2}{\left(E + \frac{W}{2} - \omega_{\vec{q}}\right)} = 0. \quad (31)$$

The population inversion can be defined as $P(t) = \langle\psi(t)|\sigma^z|\psi(t)\rangle$ [27] where $\sigma^z (= 2S^z)$ is a Pauli operator and $|\psi(t)\rangle$ is the total wavefunction in the *Schrödinger* picture, and

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}\omega_c(\frac{1}{2}\sigma^z + a^+a)t}\mathrm{e}^{-A}\mathrm{e}^{-Q}\mathrm{e}^{-\mathrm{i}H''t}\mathrm{e}^{Q}\mathrm{e}^{A}|\psi(0)\rangle. \quad (32)$$

It is reasonable to choose the initial state as $|\psi(0)\rangle = \mathrm{e}^{-A}|e\rangle|\{0_{\vec{q}}\}\rangle|\mathrm{vac}\rangle$, in which $|\mathrm{vac}\rangle$ is the vacuum state of the cavity field which corresponds to the number of the photon $m = 0$. Thus

$$P(t) = \langle\mathrm{vac}|\langle\{0_{\vec{q}}\}|\langle e|\mathrm{e}^{-Q}\mathrm{e}^{\mathrm{i}H''t}\mathrm{e}^{Q}\sigma^z\mathrm{e}^{-Q}\mathrm{e}^{-\mathrm{i}H''t}\mathrm{e}^{Q}|e\rangle \times |\{0_{\vec{q}}\}\rangle|\mathrm{vac}\rangle. \quad (33)$$

By using equations (29)–(33), we can obtain:

$$P(t) = -\frac{(1-\cos\theta)}{2}\cos\theta - \frac{(1+\cos\theta)}{2}\cos\theta \times \sum_{\vec{q},E,E'} y_{\vec{q}}(E')y^*_{\vec{q}}(E)x^*(E')x(E)e^{i(E-E')t} + \frac{(1+\cos\theta)}{2}\cos\theta\sum_{E,E'}|x(E)|^2|x(E')|^2 e^{i(E-E')t} + \frac{\sin^2\theta}{2}\sum_E(|x(E)|^2 e^{i(E+\frac{W}{2})t} + |x(E)|^2 e^{-i(E+\frac{W}{2})t}). \quad (34)$$

Employing the orthogonal property:

$$\sum_{\vec{q}} y_{\vec{q}}(E')y^*_{\vec{q}}(E) = \delta(E-E') - x(E')x^*(E) \quad (35)$$

and

$$\sum_E |x(E)|^2 e^{\pm iEt} = \frac{1}{2\pi i}e^{\mp i\frac{W}{2}t}\oint \frac{e^{\pm i(E+\frac{W}{2})t}\,dE}{E - \frac{W}{2} - \sum_{\vec{q}}\frac{V^2_{\vec{q}}}{E+\frac{W}{2}-\omega_{\vec{q}}}} = \frac{1}{2\pi i}e^{\mp i\frac{W}{2}t}\oint\frac{e^{\pm i\omega t}\,d\omega}{\omega - W - \sum_{\vec{q}}\frac{V^2_{\vec{q}}}{\omega-\omega_{\vec{q}}\mp i0^+}}, \quad (36)$$

where we have replaced $\omega$ by $E+\frac{W}{2}$, and the real and imaginary part of $\frac{V^2_{\vec{q}}}{\omega-\omega_{\vec{q}}\pm i0^+}$ as $R(\omega)$ and $\mp\gamma(\omega)$, respectively, and we obtain

$$R(\omega) = \sum_{\vec{q}}\wp\frac{V^2_{\vec{q}}}{\omega-\omega_{\vec{q}}} = -4(\eta g)^2\int_0^\infty d\omega'\frac{J(\omega')}{(\omega-\omega')(\omega'+W)}, \quad (37)$$

$$\gamma(\omega) = \pi\sum_{\vec{q}}V^2_{\vec{q}}\delta(\omega-\omega_{\vec{q}}) = 4\pi(\eta g)^2\frac{J(\omega)}{(\omega+W)^2}, \quad (38)$$

where $\wp$ represents the Cauchy principle value. $J(\omega) = \sum_{\vec{q}} M^2_{\vec{q}}\delta(\omega-\omega_{\vec{q}})$ is the spectral density. The parameter $\eta$ can be decided by equation (23) and can be expressed as

$$\eta = \exp\left\{-\int_0^\infty d\omega\frac{J(\omega)}{2(\omega+W)^2}\right\}. \quad (39)$$

The contour integral in equation (36) can be calculated by the residue theorem, and then we obtain:

$$P(t) = -\cos\theta + \cos\theta(1+\cos\theta)e^{-2\gamma(\omega_0)t} + \sin^2\theta\cos(\omega_0 t)e^{-\gamma(\omega_0)t}, \quad (40)$$

where $\omega_0$ is the solution of the equation

$$\omega - W - R(\omega) = 0, \quad (41)$$

where

$$W = \sqrt{4(\eta g)^2 + (\Delta')^2}. \quad (42)$$

As Kuhn *et al* [33, 34] have shown that at very low temperatures the deformation coupling which arises from LA-phonons makes the dominant contribution of phonons in a single quantum dot, so we use the superohmic spectral density to describe the deformation coupling: $J(\omega) = 2\alpha\omega^3 e^{-\frac{1}{2}(\frac{\omega}{\omega_l})^2}$ where $\omega_l = s/l$ is the cut-off frequency ($s$ is the sound velocity and $l$ is the dot size) and $\alpha$ is the coupling constant which can be expressed as [35, 36] $\alpha = \frac{(D_c-D_v)^2}{8\pi^2\rho s^5}$ where $D_c$ and $D_v$ are the deformation potentials for conduction band and valence band respectively, and $\rho$ refers to the material density. For the self-organized InAs/GaAs quantum dots, $D_c - D_v = 6.5$ eV, $\rho = 5.667$ g cm$^{-3}$ and $s = 3800$ m s$^{-1}$ [37–39], and then the deformation coupling constant $\alpha \approx 0.036$ ps$^2$. In the second order approximation, $\omega_0 \approx W$, and equation (38) becomes

$$\gamma = \gamma(W) = 2\pi\eta^2 g^2\alpha W e^{-\frac{1}{2}(\frac{W}{\omega_l})^2}. \quad (43)$$

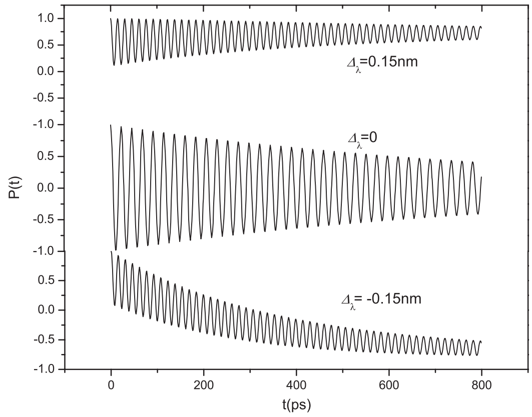


**Figure 1.** The population inversion as a function of time for three different detunings (−0.15 nm, 0, 0.15 nm). The single-photon Rabi frequency $g$ is 90 $\mu$eV, the cut-off frequency $\omega_l$ is 1 ps$^{-1}$ and the exciton–phonon coupling constant $\alpha$ is 0.036 ps$^2$.

This explicit expression is the phonon induced decoherence rate of the coupled exciton–cavity modes. It is obvious that this decoherence depends strongly on the detuning between exciton and cavity mode (see equations (24) and (43)), the strength of exciton–phonon coupling, the single-photon Rabi frequency and the size of the quantum dot. From equation (43) we can obtain the dephasing time $T_2 = \gamma^{-1}$. If the collision time is neglected, the lifetime $T_1$ of the quantum dot exciton (a two-level system) is approximately equal to $\frac{1}{2}T_2$ as shown in [31]. Recently, Hennessy *et al* [16] have measured the lifetime of quantum dot excitons for several detunings in a strongly coupled single quantum dot–cavity system, therefore we can compare our calculations to this experimental observation.

If exciton–phonon couplings in the dot–cavity system are neglected, then $\alpha = 0$ and $\gamma = 0$, we obtain from equation (40)

$$P(t) = \frac{1}{\Delta^2+4g^2}\{\Delta^2 + 4g^2\cos[(\Delta^2+4g^2)^{1/2}t]\}. \quad (44)$$

This is the vacuum Rabi oscillation with detuning $\Delta(=\omega_{ex}-\omega_c)$ in the Jaynes–Cummings model [31]. Therefore, equation (40) describes a damped vacuum Rabi oscillation in the dot–cavity system which couples to a phonon bath (also see figure 1). As the detuning increases, the amplitude of the oscillation reduces as a second order of $\sin\theta$ which is characterized by the ratio of detuning ($\Delta$) to the single-photon Rabi frequency ($g$) according to equation (40). So the detuning tends to diminish the Rabi oscillation as commonly considered. From equation (40), we can see that the second

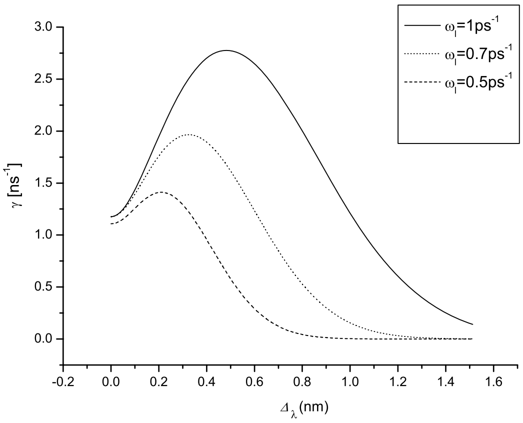


**Figure 2.** The phonon induced decoherence rate $\gamma$ as a function of detuning with three different $\omega_l$ (0.5, 0.7, 1 ps$^{-1}$). The single-photon Rabi frequency $g$ is 90 $\mu$eV and the exciton–phonon coupling constant $\alpha$ is 0.036 ps$^2$.

term, which behaves as a direct decay instead of an oscillation, apparently appears only for the non-zero detuning. If there is no detuning, the final state is found to be $P(\infty) = 0$, but is shifted to $P(\infty) = -\cos\theta$ with non-zero detuning according to equation (40). Compared to the non-oscillation term $\frac{\Delta^2}{\Delta^2+4g^2}$ in equation (44) describing the situation of non-zero detuning without exciton–phonon coupling, we find that the shift of the final state appears in terms of the joint effect of both the detuning and the phonon–exciton coupling. So the second term in equation (40) refers to the contribution of phonon–exciton coupling to the shift of the final state in the asymmetric situation with non-zero detuning. It behaves as a fast direct decay and disappears in the symmetric case without detuning. Therefore, equation (40) indicates a physical picture such that for the non-zero detuning, the excitonic Rabi oscillation emitting and absorbing a photon becomes less effective and tends to cease with increasing time because of the influence of phonon bath. The exciton–phonon coupling results in a fast decay of the whole system of the strongly coupled exciton–cavity mode, the oscillation ends up in the state $P(\infty) = -\cos\theta$.

## 3. Results and discussions

In order to compare our calculations with recent experiment [16], we choose self-organized InAs/GaAs quantum dots for illustrating the numerical results. The wavelength of the exciton is 944.8 nm and the single-photon Rabi frequency $g$ is 90 $\mu$eV. There are no exact experimental results about the size of quantum dot [40]. Here for the sake of simplicity, we assume that the InAs quantum dot is spherical, then the above exciton wavelength corresponds to a dot size of 4.2 nm. Figure 1 shows the population inversion as a function of time for three different detunings $\Delta_\lambda$ ($= \lambda_{ex} - \lambda_m$, where $\lambda_{ex}$ and $\lambda_m$ represent the resonant exciton and cavity-mode wavelength, respectively). The cut-off frequency $\omega_l$ corresponding to the dot size is assumed to be 1 ps$^{-1}$ (i.e. dot size $l \approx 4.2$ nm). It is obvious from the figure that a damped vacuum Rabi oscillation can occur in the strong coupling regime. We can also see

**Table 1.** A comparison between the experimental results [16] and the calculated values in this work.

| Calculated/experimental | $\Delta_\lambda$ (nm) | $T_1$ (ns) |
|---|---|---|
| Calculated | 0 | 0.425 |
| | 1.26 | 1.1 |
| | 1.30 | 1.4 |
| Experimental [16] | 0 | 0.06 |
| | 1.30 | 1.6 |

that the symmetry of population inversion is broken when the detuning is not equal to zero.

Figure 2 presents the decoherence rate ($\gamma$) of Rabi oscillation due to influence of the phonon bath as a function of detuning with three different $\omega_l$ (0.5, 0.7, 1 ps$^{-1}$). It shows that the decoherence rate first increases then decreases with an increase of the detuning. We can also see a fast reduction of decoherence rate with an increase of detuning as the detuning becomes large. According to our theory, when the detuning is larger, $\omega_0$ in equation (41) goes beyond the cut-off frequency which is decided by the dot size. As a result, the decoherence rate, which is deeply related to the spectral density, reduces fast and becomes very small for large detuning. Therefore, if the detuning is larger than a special value (e.g. for the size 4.2 nm of InAs quantum dot, the detuning is larger than 0.5 nm), the quantum dot exciton lifetime is increased with increased detuning. In the recent experimental conditions, we obtain that the lifetimes $T_1$ of quantum dot exciton for the detuning of 1.3 nm at $\omega_l = 1$ ps$^{-1}$ (i.e. the dot size $l \approx 4.2$ nm) is 1.4 ns, which is in good agreement with the experimental value 1.6 ns by Hennessy *et al* [16] (see table 1). From table 1, we also note that the calculated value does not agree well with the experimental result at resonance ($\Delta_\lambda = 0$). This is because the contribution from the quadratic coupling to acoustic phonons becomes significant and may not be neglected at zero detuning as discussed by Muljarov and Zimmermann [41]. As shown in the figure our theory predicts that the phonon induced decoherence rate increases in the small regime of exciton–cavity detuning, which seems a little puzzling since the excitonic-type contribution to the cavity–exciton mode is commonly regarded as diminishing with increasing exciton–cavity detuning. However, here we only take phonon–exciton interaction as the major source of decoherence and the dot size is assumed to be rather small. In such a case, the phonon–exciton coupling becomes significant and is increased with small detuning so that it causes an increase of decoherence rate. But as the dot size becomes slightly larger, the decoherence rate diminishes much faster and a further increase in value with detuning becomes much less obvious (see figure 2). This implies the significant effect of dot size on this phonon–exciton–photon coupling system and suggests that for the quantum computation based on dot–cavity system, the size of dot can be adjusted to modify the decoherence rate due to detuning. For applications in quantum computation systems based on quantum dots [42], detuning is usually used to prevent some transitions, so a good understanding of detuning effects helps one to control the process better.

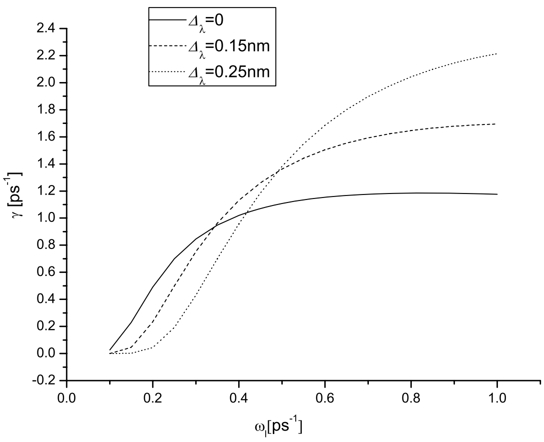


**Figure 3.** The phonon induced decoherence rate $\gamma$ as a function of the cut-off frequency $\omega_l$ with three different detunings (0, 0.15 nm, 0.25 nm). The single-photon Rabi frequency $g$ is 90 $\mu$eV and the exciton–phonon coupling constant $\alpha$ is 0.036 ps$^2$.

Figure 3 shows the decoherence rate as a function of the cut-off frequency ($\omega_l$) with three different detunings (0, 0.15 nm, 0.25 nm). We can see that the decoherence rate initially increases with the cut-off frequency, changing fairly smoothly as the cut-off frequency becomes large. This shows the impact of the dot size on the phonon induced decoherence rate, and when the dot size becomes small (i.e. for large $\omega_l$), the variation of decoherence rate via the dot size is less obvious. It should be emphasized that many other factors may also cause decoherence, such as the coupling with the continuum wetting layer states or the existence of multiexciton states [43]. The results including these factors are certainly complete. We will treat these factors in our theory elsewhere.

## 4. Conclusions

In conclusion, we have investigated the detuning effect on quantum dynamics and phonon induced decoherence of a single quantum dot–cavity system based on a unitary transformation and an operator displacement. A damped vacuum Rabi oscillation in the strong coupling regime is presented. A rather simple form of the decoherence induced by acoustic phonons as a function of the detuning between the cavity mode and exciton is obtained explicitly and a comparison with recent experimental observations is demonstrated. The good agreement between the model and the experimental results confirms our theoretical method proposed here is reasonable. It should be noted that our model calculation does not agree with the experimental result at very large detuning ($\Delta_\lambda = 4.1$ nm) since the proposed model is no longer suitable at large detunings. Finally, we hope that the predictions of this work can be confirmed by experiment in the near future and may offer improvements in quantum information processing based on quantum dot–cavity systems.

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